Cross products

Review determinants

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\left|\begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix}\right| \begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix}\Bigg|$$

$$= aei + bfg + cdh$$
$$- ceg - bdi - afh$$

$$\langle x, y, z\rangle \times \langle u, v, w\rangle$$

$$(xi + yj + zk) \times (ui + vj + wk)$$

$$= \begin{vmatrix} i & j & k \\ x & y & z \\ u & v & w \end{vmatrix} = (yw - zv)i$$
$$- (xw - zu)j$$

$+ (xv - yu)k$

$$\left|\begin{array}{ccc|ccc} i & j & k & i & j & k \\ x & y & z & x & y & z \\ u & v & w & u & v & w \end{array}\right|$$

$$\langle \underset{x}{5}, \underset{y}{6}, \underset{z}{2} \rangle \times \langle \underset{u}{2}, \underset{v}{0}, \underset{w}{2} \rangle$$

$$= 12i - 6j - 12k$$

$u \times v \neq v \times u$ no!!!

Magnitude

$|a \times b|$ = area of the parallelogram formed by a & b

$= |a||b| \sin\theta$

$|a \times b| = |a||b| \sin\theta$

= 2(area of triangle

formed by a & b)

* $a \times b = -(b \times a)$

Ex: $u \times v$ & $v \times u$

if $u = 2i + j + K$

& $V = -4i + 3j + K$

$$\begin{vmatrix} i & j & K \\ 2 & 1 & 1 \\ -4 & 3 & 1 \end{vmatrix} = (1-3)i - (2+4)j + (6+4)K$$

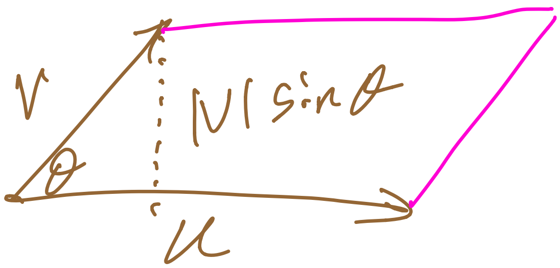
$$= -2i - 6j + 10K$$

$u \times V$ ↗

$V \times u = 2i + 6j - 10K$

Area of parallelogram

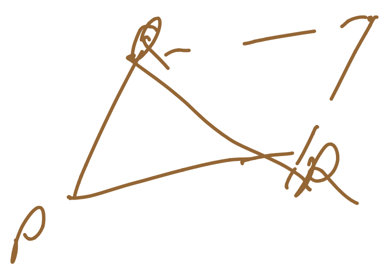
If we locate vectors u & v s.t. they form adjacent sides of a parallelogram, then the area of the parallelogram is given by $|u \times v|$



Find the area of a triangle

$P = (1,0,0)$ $\quad Q = (0,1,0)$

& $R = (0,0,1)$



$\overrightarrow{PQ} = \langle -1, 1, 0 \rangle$

$\overrightarrow{PR} = \langle -1, 0, 1 \rangle$

$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} i & j & k \end{vmatrix}$ $\quad (1-0)\, i$

$$= -(-1-0)j + (0+1)K$$

$$= i + j + K$$

$$\left|\vec{PQ} \times \vec{PR}\right| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$$

Area of a triangle is

$$\sqrt{3}/2$$

Find an Orthogonal Vector

Only a single plane can pass through any set of 3 non colinea points.

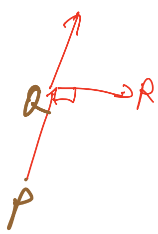
Find a vector orthogonal to the plane containing points

$P = (9, -3, 2)$, $Q = (1, 3, 0)$

& $R = \langle -2, 4, 0 \rangle$

The plane must contain vectors

$\overrightarrow{PQ}$ & $\overrightarrow{QR}$



$$\overrightarrow{PQ} = \langle -8, 6, -2 \rangle$$

$$\overrightarrow{QR} = \langle -3, 2, 0 \rangle$$

$$\overrightarrow{PQ} \times \overrightarrow{QR} = \begin{vmatrix} i & j & k \\ -8 & 6 & -2 \\ -3 & 2 & 0 \end{vmatrix}$$

$$= {}^{+}(0+4)i - (0-(+6))j \overset{+}{+} (-16+18)k$$

$$= 4i + 6j + 2k$$